# High-Throughput Reliable Multicast in Multi-Hop Wireless Mesh Networks

Xin Zhao, *Member, IEEE*, Jun Guo, *Member, IEEE*, Chun Tung Chou, *Member, IEEE*, Archan Misra, *Member, IEEE*, and Sanjay K. Jha, *Senior Member, IEEE*

**Abstract—**This paper presents a cross-layer approach for enabling high-throughput reliable multicast in multi-hop wireless mesh networks. The building block of our approach is a multicast routing metric, called the *expected multicast transmission count* (EMTX). EMTX is designed to capture the combined effects of MAC-layer retransmission-based reliability, wireless broadcast advantage, and link quality awareness. The EMTX of single-hop transmission of a multicast packet from a sender is the expected number of multicast transmissions (including retransmissions) required for its next-hop recipients to receive the packet successfully. We formulate the EMTX-based multicast problem with the objective of minimizing the sum of EMTX over all forwarding nodes in the multicast tree, aiming to reduce network bandwidth consumption while ensure high end-to-end packet delivery ratio for the multicast traffic. We provide rigorous mathematical formulations and methods to find near-optimal solutions of the problem computationally efficiently. We present centralized and distributed algorithms, and demonstrate their effectiveness in tackling the EMTX-based multicast problem with a combination of theoretical and numerical results. Simulation experiments show that, in comparison with two baseline approaches, EMTX-based multicast routing reduces the number of hop-by-hop transmissions per packet by up to 40 percent and yet improves the multicast throughput by up to 24 percent.

**Index Terms—**Wireless mesh network, multicast algorithm, routing metric, cross-layer design

## 1 Introduction

Multicast is an important transmission mechanism commonly defined in wireless networking standards. Due to the broadcast nature of wireless communications, a single transmission using the wireless medium can reach multiple nodes within the transmission range of the sender. In multi-hop wireless networks such as wireless mesh networks [1] where the capacity can often be very limited [2], it is desirable to exploit the *wireless broadcast advantage* in the design of multicast routing algorithms to support various applications involving group communication.

The widely deployed IEEE 802.11 standard defines a basic multicast mechanism [3]. In the case of a sender multicasting to multiple nodes, the MAC-layer protocol chooses the lowest available transmission rate by default and does not provide any MAC-layer error recovery. However, communications over the wireless medium are by nature error-prone. Significant variations in fading and interference levels may lead to transient loss of a link. In multi-hop wireless networks, packet collisions caused by hidden nodes often result in added packet loss. This can make the end-to-end packet delivery ratio (or throughput) unacceptable for many applications, including multicast, that require reliable data transmissions in wireless mesh networks.

In the literature, a number of reliable MAC-layer multicast mechanisms were proposed to overcome the inefficiency of the IEEE 802.11 standard [4], [5], [6], [7], [8], [9], [10]. They are designed in various ways to provide explicitly hop-by-hop recovery on packet loss. A handful of researchers explored the idea of *physical-layer network coding* and developed bandwidth efficient methods for link-layer acknowledgement for multicast transmissions in wireless networks [11], [12], [13].

Since mesh routers in wireless mesh networks are in general stationary and do not have energy constraint typical of ad hoc networks, a key objective of wireless mesh networks is to offer high-throughput wireless connection to end users. Despite active research on reliable MAC-layer multicast for single-hop group communication, it is challenging to develop high-throughput algorithms for reliable multicast routing in multi-hop wireless networks. This paper addresses this challenge by proposing a robust multicast routing metric, called the *expected multicast transmission count* (EMTX). A preliminary study of EMTX-based multicast routing was presented in [14]. Here, we provide a more thorough understanding of the metric, and systematically design and evaluate routing algorithms based on the metric for enabling high-throughput reliable multicast in multi-hop wireless mesh networks.

The main contributions of this paper are:

- We propose EMTX as a metric for achieving high-throughput reliable multicast routing in multi-hop wireless mesh networks. EMTX is explicitly designed to capture the combined effects of 1) MAC-layer retransmission-based reliability, 2) wireless


- X. Zhao is with the Australian Centre for Field Robotics, The University of Sydney, Australia. E-mail: x.zhao@acfr.usyd.edu.au.
- J. Guo is with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong. E-mail: j.guo@cityu.edu.hk.
- C.T. Chou and S.K. Jha are with the School of Computer Science and Engineering, University of New South Wales, Australia. E-mail: {ctchou, sanjay}@cse.unsw.edu.au.
- A. Misra is with the School of Information Systems, Singapore Management University, Singapore E-mail: archanm@smu.edu.sg.


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broadcast advantage and 3) link quality awareness. We discuss and rigorously prove the important properties of EMTX, and provide a method for reducing the complexity of computing the metric.

- We formulate the EMTX-based multicast problem with the objective of minimizing the sum of EMTX over all forwarding nodes in the multicast tree. We prove that the problem is NP-hard. A mathematical formulation of the problem in the form of integer linear programming (ILP) is provided. Based on the ILP formulation, we show how to solve the optimization problem computationally efficiently by using the Lagrangian relaxation technique.
- We present a polynomial-time greedy algorithm for the multicast problem and analyze its worst-case approximation ratio. Further, we extend the centralized algorithm to a distributed version as an EMTX-based multicast routing protocol.

The remainder of this paper is organized as follows. In Section 2, we review the literature regarding multicast in wireless ad hoc networks and discuss the motivation of our work. Section 3 presents the detailed design and important properties of the EMTX metric. Section 4 formulates the EMTX-based multicast problem. Lagrangian relaxation is provided in Section 5. The centralized algorithm and its distributed version are presented in Sections 6 and 7, respectively. Description of protocol implementation and its evaluation are provided in Section 8. Finally, Section 9 draws the conclusion.

## 2 Related Work

### 2.1 Reliable MAC-Layer Multicast

Current IEEE 802.11 standards do not support MAC-layer error recovery for multicast transmissions. With the aim of improving the reliability of single-hop multicast transmissions, a number of reliable MAC-layer multicast protocols were proposed in the literature. We provide a quick summary of some of these schemes with a view to use one of these for our simulations.

One method is ARQ-based MAC-layer multicast by extending the RTS/CTS/ACK control frames of IEEE 802.11 MAC [4], [5]. The leader-based protocol presented in [4] avoids the need of multiple positive ACK frames for multicast transmissions. The batch mode multicast MAC protocol presented in [5] uses a strict sequential order of RTS/CTS to each destination. The HIMAC solution proposed in [6] uses unary channel feedback and unary negative feedback to address two shortcomings in 802.11 multicast: channel-state indifference and demand ignorance. In [7], Kim et al. used the direct-sequence code-division multiple access scheme, which allows multiple receivers to transmit ACK frames concurrently to reduce the overhead. In [8], Kim et al. used the orthogonal frequency-division multiple access mechanism to deal with the overhead issue of ARQ-based MAC-layer multicast protocols. Both the 802.11 MX protocol presented in [9] and the RMAC protocol presented in [10] use the busy tone mechanism to offer reliable MAC-layer multicast. Relying on a separate channel, busy tone prevents data frame collisions and solves the hidden terminal problem. Although busy tone is a technique different to ARQ, they both share the same core idea: acknowledgement and retransmissions.

The EMTX-based multicast routing protocol proposed in this paper can be implemented over any of these single-hop MAC-layer multicast protocols. In this paper, we use RMAC in our simulations as the reliable MAC-layer multicast protocol.

### 2.2 Multicast Routing Metric

As discussed earlier, given that the mesh routers are not energy constrained, the design objective is to provide high-performance wireless connection to end users. This motivates the design of robust routing metrics that can find high-performance paths compared to the simple hop-count metric or minimum-energy routing used in most ad hoc and sensor networks [15], [16], [17].

The expected transmission count (ETX) metric proposed in [18] is a popular link-quality-aware metric designed for multi-hop wireless routing in unicast applications. ETX aims to find high-throughput paths that minimize the expected total number of MAC-layer transmissions (including retransmissions) required for delivering a packet hop-by-hop to its destination. One can apply ETX to form a shortest path tree (SPT), which aims to find a high-throughput unicast path for each multicast destination. However, such a scheme of multiple unicasts cannot fully utilize the wireless broadcast advantage and thus may consume excessive network bandwidth. In this paper, we use ETX-based SPT as one baseline approach for demonstrating the effectiveness of EMTX-based multicast routing.

The multicast routing metric considered in [19] aims to form a multicast tree that minimizes the number of forwarding nodes. It is based on a binary packet reception model, which implicitly assumes that transmissions on each link in the wireless network are 100 percent reliable. It makes use of the wireless broadcast advantage but ignores the link quality. The EMTX metric proposed in this paper captures the effects of unreliable wireless links, which is true for most wireless networks. In this paper, we call the approach of [19] as the minimum forwarder tree (MFT). We show that MFT is a special case of our EMTX-based multicast approach and use it as another baseline approach for performance comparison.

The multicast routing metrics proposed in [20] and [21] are similar in that, for a single-hop multicast transmission, they use the amount of ETX needed for the receiver with the worst link quality to approximate the expected number of multicast transmissions required for all receivers to receive the packet successfully. The computation of the EMTX metric proposed in this paper for a single-hop multicast transmission takes as input the link quality from the sender to each receiver.

Roy et al. [22] studied several routing metrics for high-throughput multicast in wireless mesh networks. They are all based on the legacy multicast mechanism defined in IEEE 802.11 standards, and do not take MAC-layer retransmission into account. The EMTX metric that we propose in this paper is designed to take advantage of MAC-layer retransmission-based reliability.

### 2.3 Multicast Routing Algorithm

Various multicast routing algorithms were proposed for wireless mesh networks. Chou et al. [23] and Liu et al. [24] studied the problem of maximizing multicast traffic load in wireless mesh networks, and presented a number of algorithms for achieving low latency multicast using the wireless broadcast advantage and multi-rate radios. In [25], a resilient forwarding mesh (RFM) approach is proposed for protecting multicast sessions from link or node failures. The optimal RFM is a set of forwarding nodes that form a pair of node-disjoint paths for each multicast destination, minimizing the number of broadcast transmissions by exploiting the wireless broadcast advantage. The source-initiated wireless multicast algorithm proposed in [26] constructs a shared tree on which each multicast destination has the minimum possible depth (number of hops from the nearest source).

A common assumption of this related work is that mesh routers in wireless mesh networks follow the binary packet reception model, which fails to take wireless link quality into account. To the best of our knowledge, there is no multicast routing algorithm explicitly designed for high-throughput reliable multicast routing in wireless mesh networks. In this paper, we propose both centralized and distributed algorithms for EMTX-based multicast routing, which effectively take into account MAC-layer retransmission and wireless link quality.

### 2.4 Network Coding

An alternative approach to realise efficient multicast is via network coding. Kim et al. [27] presented an algorithm to compute the network code that can be used to realise layered multicast; such code would be especially useful if the multicast receivers have heterogeneous requirements on video quality. Vieira et al. [28] studied how link-layer multirate multicast can be used in conjunction with network coding to increase network-layer multicast throughput. On the subject of reliable broadcast, Li et al. [29] studied the use of immediately decodable network codes to realise reliable broadcast of stored videos in the presence of packet loss and time delay constraint. Such codes are hard to analyse and the authors proved that asymptotic throughput is achieved for two-user and three-user cases if the size of the stored video is sufficiently large. Finally, Ghaderi et al. [30] have compared, purely by a theoretical analysis, the relative performance of network coding versus hop-by-hop ARQ as a means to achieve reliable multicast for a special full $K$-ary tree, but did not consider the optimal choice of the multicast tree (which is our focus in this paper). It would be interesting to study the performance of such codes in practical scenarios and compare with the approach in this paper. However, all such network coding approaches focus primarily on improving throughput or ensure only probabilistic (and thus non-guaranteed) delivery to all receivers. In contrast, we focus in this paper on designing multicast trees that provide guaranteed delivery of multicast traffic while optimizing the total transmission overhead.

## 3 Metric Design

This section presents the detailed design and properties of the EMTX metric for multicast routing in multi-hop wireless mesh networks. As discussed, the EMTX metric is designed to capture the combined effects of 1) MAC-layer retransmission-based reliability, 2) wireless broadcast advantage and 3) link quality awareness. The computation of EMTX for a single-hop multicast transmission takes as input the *link quality* from the sender to each of its next-hop receivers. In this paper, we use the term link quality with a specific meaning from this point onward. We define the link quality of the (directed) wireless link $\langle i, j\rangle$ from node $i$ to node $j$ as the probability that a multicast transmission from node $i$ is successfully received and acknowledged by node $j$. Note that the link qualities of $\langle i, j\rangle$ and $\langle j, i\rangle$ are not necessarily the same because the delivery probabilities for data and ACK frames can be different in the directions $\langle i, j\rangle$ and $\langle j, i\rangle$. In our multicast framework, MAC-layer retransmission is used for reliability. This means that a sender will retransmit a multicast packet to its next-hop receivers which have not acknowledged the packet successfully. The EMTX of single-hop transmission of a multicast packet is defined as the expected number of multicast transmissions needed for all next-hop recipients to receive and acknowledge the packet successfully including retransmissions.

For the purpose of this section, it suffices to consider one sending node $i$ in the network and the set of its next-hop receivers $\mathcal{R}_i$ within its single-hop *neighborhood* $N_i$. The single-hop neighborhood of node $i$ is defined as the set of nodes within the transmission range of node $i$. For node $j$ to be within the transmission range of node $i$, we require that the link quality of the wireless link $\langle i, j\rangle$, denoted by $p_{i,j}$, is non-zero.

In computing EMTX, we assume that the MAC layer of the sender continues to retransmit the multicast packet until it is successfully received and acknowledged by each of its next-hop receivers. We also assume that the link quality from the sender to each of its next-hop receivers is spatially and temporally independent. These assumptions are widely accepted in the literature, such as [31], [32]. In this manner, the sequence of transmission/retransmissions of the multicast packet forms independent Bernoulli trials with an identical success probability.

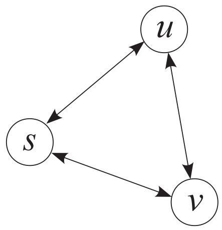
Let $EMTX_{i,\mathcal{R}_i}$ denote the amount of EMTX required from node $i$ to all nodes in $\mathcal{R}_i$. Let $\Upsilon(\mathcal{R}_i, c)$, $c = 1, 2, \ldots, |\mathcal{R}_i|$, denote the set of unordered choices of $c$ elements from $\mathcal{R}_i$. Let $f_{i,j} = 1 - p_{i,j}$. Theorem 1 below derives a closed-form expression for computing $EMTX_{i,\mathcal{R}_i}$ based on the definition of EMTX and its assumptions.

**Theorem 1.** *$EMTX_{i,\mathcal{R}_i}$ is given by*

$$EMTX_{i,\mathcal{R}_i} = \sum_{c=1}^{|\mathcal{R}_i|} (-1)^{c-1} \sum_{\mathcal{S} \in \Upsilon(\mathcal{R}_i, c)} \frac{1}{1 - \prod_{j \in \mathcal{S}} f_{i,j}}. \quad (1)$$

**Proof.** See Appendix A in the supplementary file, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2014.2333731. □

For the three-node wireless mesh network example provided in Fig. 1, let us consider node $s$ sending a multicast packet to both node $u$ and node $v$. Based on the




| $\langle i,j \rangle$ | $p_{i,j}$ |
|---|---|
| $\langle s,u \rangle$ | 0.8 |
| $\langle u,s \rangle$ | 0.9 |
| $\langle s,v \rangle$ | 0.7 |
| $\langle v,s \rangle$ | 0.6 |
| $\langle u,v \rangle$ | 0.6 |
| $\langle v,u \rangle$ | 0.7 |

Fig. 1. A three-node wireless mesh network example.

$p_{i,j}$ values provided in Fig. 1, the calculation of EMTX using (1) shows that

$$\text{EMTX}_{s,\{u,v\}} = \frac{1}{1-0.2} + \frac{1}{1-0.3} - \frac{1}{1-0.2\times 0.3} = 1.61.$$

We know from [18] that $\text{ETX}_{s,u} = 1/(1-0.2) = 1.25$ and $\text{ETX}_{s,v} = 1/(1-0.3) = 1.43$. These results indicate that

$$\text{EMTX}_{s,\{u,v\}} < \text{ETX}_{s,u} + \text{ETX}_{s,v}$$

and

$$\text{EMTX}_{s,\{u,v\}} > \max(\text{ETX}_{s,u}, \text{ETX}_{s,v}).$$

Intuitively, with the wireless broadcast advantage, the amount of EMTX that the sender requires for a multicast packet must be smaller than the total amount of ETX required when the packet is unicast to each of its next-hop receivers, and must also be larger than the amount of ETX needed for its next-hop receiver with the worst link quality. We summarize and prove the properties of EMTX in a more general form as shown below.

**Theorem 2.** *For node $i \in V$, consider two subsets $\mathcal{R}_1 \subseteq N_i$ and $\mathcal{R}_2 \subseteq N_i$. If $\mathcal{R}_1 \subseteq \mathcal{R}_2$, we have*

$$EMTX_{i,\mathcal{R}_1} \leq EMTX_{i,\mathcal{R}_2}.$$

**Proof.** See Appendix B available in the online supplemental material. □

**Theorem 3.** *For node* i $\in V$, *consider two subsets $\mathcal{R}_1 \subseteq N_i$ and $\mathcal{R}_2 \subseteq N_i$. Then,*

$$EMTX_{i,\mathcal{R}_1 \cup \mathcal{R}_2} \leq EMTX_{i,\mathcal{R}_1} + EMTX_{i,\mathcal{R}_2}.$$

**Proof.** See Appendix C available in the online supplemental material. □

**Corollary 1.** *For $|\mathcal{R}_i| > 1$, $EMTX_{i,\mathcal{R}_i}$ has the lower and upper bounds given by*

$$\max_{j\in\mathcal{R}_i} ETX_{i,j} \leq EMTX_{i,\mathcal{R}_i} < \sum_{j\in\mathcal{R}_i} ETX_{i,j}. \tag{2}$$

Note that the equivalence to the lower bound in (2) holds only when $p_{i,j} = 1$ for all $j \in \mathcal{R}_i$.

**Remark 1.** The inequality on the right-hand side of (2) captures the wireless broadcast advantage of using EMTX. The difference between $\text{EMTX}_{i,\mathcal{R}_i}$ and $\sum_{j\in\mathcal{R}_i} \text{ETX}_{i,j}$ is in fact the number of transmissions that can be saved by using multicast instead of unicast. We will see in Section 8 that EMTX is effective in reducing the number of transmissions in a network setting compared to ETX. However, in terms of algorithm design, the use of EMTX as a routing metric is more difficult than that of using ETX because the optimization problem is NP-hard, as we will see in Section 4.

### 3.1 Reducing Computational Complexity

Note that the complexity of computing the $\text{EMTX}_{i,\mathcal{R}_i}$ value for any $(i,\mathcal{R}_i)$ pair in (1) is exponential in the size of $\mathcal{R}_i$. Below, we design an alternative way for more efficiently computing $\text{EMTX}_{i,\mathcal{R}_i}$, especially when the size of $\mathcal{R}_i$ is large.

Without loss of generality, we label nodes in the set $\mathcal{R}_i$ in a non-increasing order of their $f_{i,j}$ values, i.e., $\mathcal{R}_i = \{1,2,\ldots,|\mathcal{R}_i|\}$ where $f_{i,1} \geq f_{i,2} \geq \cdots \geq f_{i,|\mathcal{R}_i|}$. Define $\mathcal{R}_{i,j}$ as the subset of $\mathcal{R}_i$ given by $\mathcal{R}_{i,j} = \{1,2,\ldots,j\}$.

**Theorem 4.** *An alternative expression of* $\text{EMTX}_{i,\mathcal{R}_i}$ *is*

$$EMTX_{i,\mathcal{R}_i} = \sum_{j=1}^{|\mathcal{R}_i|} \Gamma_{i,j},$$

*where*

$$\Gamma_{i,j} = \sum_{k=1}^{\infty} f_{i,j}^k \prod_{u\in\mathcal{R}_{i,j-1}} (1 - f_{i,u}^k). \tag{3}$$

**Proof.** See Appendix D available in the online supplemental material. □

**Theorem 5.** *For an arbitrarily small value $\epsilon > 0$, there exists a positive integer $K_j$ for each $j \in \mathcal{R}_i$ such that*

$$\sum_{k=K_j}^{\infty} f_{i,j}^k \prod_{u\in\mathcal{R}_{i,j-1}} (1 - f_{i,u}^k) \leq \frac{\epsilon}{|\mathcal{R}_i|}. \tag{4}$$

**Proof.** See Appendix E available in the online supplemental material. □

Theorems 4 and 5 allow us to compute an approximation of $\text{EMTX}_{i,\mathcal{R}_i}$ by finding the smallest positive integer $K_j$ that satisfies (4) and then truncating the right-hand side of (3) at $k = K_j - 1$ for each $j \in \mathcal{R}_i$. Specifically, letting

$$\widehat{\Gamma}_{i,j} = \sum_{k=1}^{K_j-1} f_{i,j}^k \prod_{u\in\mathcal{R}_{i,j-1}} (1 - f_{i,u}^k)$$

we can obtain $\text{EMTX}_{i,\mathcal{R}_i}$ as

$$EMTX_{i,\mathcal{R}_i} = \sum_{j=1}^{|\mathcal{R}_i|} \widehat{\Gamma}_{i,j} \tag{5}$$

where it is clear that

$$\sum_{j=1}^{|\mathcal{R}_i|} \left(\Gamma_{i,j} - \widehat{\Gamma}_{i,j}\right) \leq \sum_{j=1}^{|\mathcal{R}_i|} \frac{\epsilon}{|\mathcal{R}_i|} = \epsilon.$$

**Theorem 6.** *The complexity of computing the right-hand side of (5) is $\mathcal{O}(|\mathcal{R}_i|^2 |\log(\epsilon)| + |\mathcal{R}_i|^2 \log|\mathcal{R}_i|)$.*

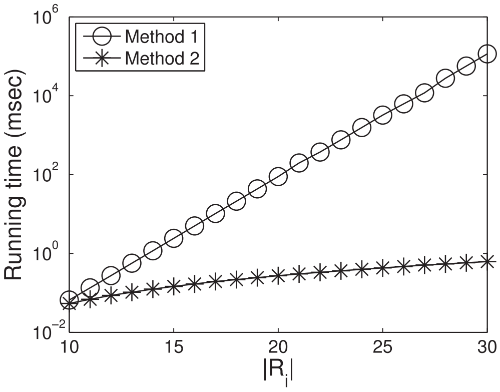



Fig. 2. Efficiency of computing $\mathrm{EMTX}_{i,\mathcal{R}_i}$.

**Proof.** See Appendix F available in the online supplemental material. □

Fig. 2 compares the running times of the two methods in computing $\mathrm{EMTX}_{i,\mathcal{R}_i}$, where Method 1 uses (1), and Method 2 uses (5) with $\epsilon$ in (4) set to 0.00001. The size of $\mathcal{R}_i$ is varied from 10 to 30. For each choice of $|\mathcal{R}_i|$, we conduct 1,000 experiments, each with a random configuration of the $f_{i,j}$ values chosen uniformly from the range $[0.1, 0.9]$. CPU running times on a 3.0 GHz Xeon machine are plotted on a logarithmic scale in Fig. 2. The results demonstrate that Method 2 is significantly more efficient than Method 1, especially when the size of $\mathcal{R}_i$ is large. For $|\mathcal{R}_i| = 30$, one computation of $\mathrm{EMTX}_{i,\mathcal{R}_i}$ using Method 1 needs nearly two minutes on average, but Method 2 requires merely 0.6 msec.

**Remark 2.** For dense networks where each node has a large number of neighbors, the network may also use topology control to reduce the number of neighbors in order to reduce the computation burden. The effect of using topology control on multicast routing performance is an interesting problem and is left for future work.

## 4 EMTX-Based Multicast Problem

In this section, we formulate the EMTX-based multicast problem. We begin by describing the model of the wireless mesh network considered in this paper, and then provide the definition of the EMTX-based multicast problem. We prove the NP hardness of the problem, and then present a mathematical formulation in the form of ILP for the problem.

### 4.1 Network Model

The wireless mesh network considered in this paper supports link-layer acknowledgement for multicast transmissions. The network is represented by a directed graph $G = (V, E)$, where $V$ is the set of mesh routers and $E$ is the set of directed links. A directed link $\langle i, j\rangle$ from node $i$ to node $j$ exists if node $j$ is within the transmission range of node $i$. As defined in Section 3, this requires $p_{i,j} > 0$. By definition, we have $p_{i,j} = \overrightarrow{d_{i,j}} \times \overleftarrow{d_{i,j}}$, where $\overrightarrow{d_{i,j}}$ and $\overleftarrow{d_{i,j}}$ are the forward delivery probability for data frames and the reverse delivery probability for ACK frames, respectively. The set of nodes $\{j : \langle i, j\rangle \in E\}$ forms the single-hop neighborhood $N_i$ of node $i$. Each node is equipped with one radio, with all radios tuned to a common channel.

### 4.2 Problem Statement

The EMTX-based multicast problem is defined for one single multicast session in the wireless mesh network. Members of the *multicast group* include one source node $s$ and a set of destination nodes $\mathcal{D}$. The problem requires to establish a directed tree $T$ of $G$ rooted at the source node and connecting all destination nodes in the multicast group. Since it is a multicast session, extra nodes may be selected from the set $V - \{s\} - \mathcal{D}$ and included in $T$ as forwarding nodes, for ensuring end-to-end connectivity and for achieving the specified optimality criterion. In its graph representation, all forwarding nodes of the multicast session (including the source node) form the set of internal nodes of $T$. Note that the internal nodes of $T$ may include certain destination nodes if they are also selected as forwarding nodes in the multicast session, but the leaf nodes of $T$ are exclusively composed of destination nodes. For convenience, we let $\mathcal{I}(T)$ denote the set of internal nodes of $T$.

Recall that the EMTX of single-hop transmission from each particular forwarding node in the multicast tree is the expected number of wireless transmissions (including retransmissions) required for delivering the multicast packet successfully to all next-hop receivers of the sender. The objective of the EMTX-based multicast problem is to find the optimal $T$ for the multicast session that yields the minimum sum of EMTX over all forwarding nodes in the set $\mathcal{I}(T)$. Since each additional multicast transmission consumes extra network bandwidth, by optimizing multicast routing in this way, we expect to reduce the total number of transmissions for the multicast session and thus increase the network throughput, while at the same time we ensure high end-to-end packet delivery ratio for the multi-hop multicast transmission.

**Theorem 7.** *The EMTX-based multicast problem is NP-hard.*

**Proof.** See Appendix G available in the online supplemental material. □

### 4.3 Mathematical Formulation

Define:

- The binary variables $e_{v,i,j}$, $\langle i, j\rangle \in E$, $v \in \mathcal{D}$, given by

$$e_{v,i,j} = \begin{cases} 1 & \text{if the directed link } \langle i,j\rangle \text{ isused} \\ & \text{by the path from the source node} \\ & \text{to the destination node } v \\ 0 & \text{otherwise.} \end{cases}$$

- The binary variables $t_{i,j}$, $\langle i, j\rangle \in E$, given by

$$t_{i,j} = \begin{cases} 1 & \text{if the directed link } \langle i,j\rangle \text{ is included} \\ & \text{in them ulticast tree} \\ 0 & \text{otherwise.} \end{cases}$$

- The binary variables $x_{i,\mathcal{R}_i}$, $i \in V$, $\mathcal{R}_i \subseteq N_i$, given by

$$x_{i,\mathcal{R}_i} = \begin{cases} 1 & \text{if node } i \text{ is selected as a forwarding} \\ & \text{node and } \mathcal{R}_i \text{ is the set of next-hop} \\ & \text{receivers selected for node } i \\ 0 & \text{otherwise.} \end{cases}$$

Then, an ILP formulation of the EMTX-based multicast problem is:

$$Z = \min \sum_{i \in V} \sum_{\mathcal{R}_i \subseteq N_i} \mathrm{EMTX}_{i,\mathcal{R}_i} \cdot x_{i,\mathcal{R}_i} \tag{6}$$

subject to

$$\sum_{j:\langle s,j \rangle \in E} e_{v,s,j} - \sum_{j:\langle j,s \rangle \in E} e_{v,j,s} = 1, \quad \forall v \in \mathcal{D} \tag{7}$$

$$\sum_{j:\langle v,j \rangle \in E} e_{v,v,j} - \sum_{j:\langle j,v \rangle \in E} e_{v,j,v} = -1, \quad \forall v \in \mathcal{D} \tag{8}$$

$$\sum_{j:\langle i,j \rangle \in E} e_{v,i,j} - \sum_{j:\langle j,i \rangle \in E} e_{v,j,i} = 0, \quad \forall v \in \mathcal{D}, \quad i \in V - \{s, v\} \tag{9}$$

$$e_{v,i,j} \leq t_{i,j}, \quad \forall v \in \mathcal{D}, \quad \langle i,j \rangle \in E \tag{10}$$

$$t_{i,j} \leq \sum_{\mathcal{R}_i : \mathcal{R}_i \subseteq N_i, j \in \mathcal{R}_i} x_{i,\mathcal{R}_i}, \quad \forall \langle i,j \rangle \in E \tag{11}$$

$$\sum_{\mathcal{R}_i \subseteq N_i} x_{i,\mathcal{R}_i} \leq 1, \quad \forall i \in V. \tag{12}$$

Constraints in (7)-(9) enforce the *flow conservation law* along the path from the source node $s$ to each destination node $v$ in the set $\mathcal{D}$. These are standard flow conservation constraints for directed graphs (see e.g. [33]). Note that although these constraints allow both $e_{v,i,j}$ and $e_{v,j,i}$ to be 1, such feasible solutions (which have circular flows) cannot be optimal because our solution is to minimise the objective in (6). Constraints in (10) ensure that the directed link $\langle i,j \rangle$ is included in the multicast tree if it is used by at least one of the end-to-end paths. Constraints in (11) ensure that, if the directed link $\langle i,j \rangle$ is included in the multicast tree, node $i$ is selected as a forwarding node, and node $j$ is one of the next-hop receivers of node $i$. Constraints in (12) ensure that, if node $i$ is selected as a forwarding node, $\mathcal{R}_i$ identifies the (unique) set of next-hop receivers of node $i$ in the multicast tree. These constraints together with the objective function in (6) jointly ensure that the optimal solution is a directed multicast tree rooted at the source node, connecting all destination nodes in the multicast group, and minimizing the sum of EMTX over all internal nodes of the tree.

## 5 Lagrangian Relaxation

The mathematical formulation of the EMTX-based multicast problem presented in Section 4.3 can be used by an ILP solver, e.g. CPLEX [34], to find optimal solutions to the problem in principle. However, ILP requires an exponentially growing computational effort as the size of the problem instances increases.

Lagrangian relaxation is a well established and more computationally efficient method for solving ILP problems [35]. It works in particular for ILP problems that can be viewed as easy problems except for certain complicating constraints. Dualizing the complicating constraints yields a Lagrangian problem that is easy to solve. In general, the solutions to the Lagrangian problem are infeasible solutions to the original problem. However, the optimal value of the Lagrangian problem defines a lower bound (for minimization problems) on the optimal value of the original problem. In contrary, feasible solutions to the original problem produce an upper bound (for minimization problems) on the optimal value of the original problem. Hence, the goal of Lagrangian relaxation is to narrow the gap between the lower bound and the upper bound.

### 5.1 The Relaxation

Consider the relaxation of the ILP formulation of the EMTX-based multicast problem by dualizing the flow conservation constraints in (7)-(9) with a set of Lagrange multipliers $\boldsymbol{\lambda} = \{\lambda_{v,i} : v \in D, i \in V\}$. Then, for a given $\boldsymbol{\lambda}$, it requires to solve the Lagrangian problem (denoted by LR) with the objective function

$$\begin{aligned} Z_{\mathrm{LR}}(\boldsymbol{\lambda}) = \min & \sum_{i \in V} \sum_{\mathcal{R}_i \subseteq N_i} \mathrm{EMTX}_{i,\mathcal{R}_i} \cdot x_{i,\mathcal{R}_i} \\ & + \sum_{v \in D} \lambda_{v,s} \Bigg( \sum_{j:\langle s,j \rangle \in E} e_{v,s,j} - \sum_{j:\langle j,s \rangle \in E} e_{v,j,s} - 1 \Bigg) \\ & + \sum_{v \in D} \lambda_{v,v} \Bigg( \sum_{j:\langle v,j \rangle \in E} e_{v,v,j} - \sum_{j:\langle j,v \rangle \in E} e_{v,j,v} + 1 \Bigg) \\ & + \sum_{v \in D} \sum_{i \in V - \{s,v\}} \lambda_{v,i} \Bigg( \sum_{j:\langle i,j \rangle \in E} e_{v,i,j} - \sum_{j:\langle j,i \rangle \in E} e_{v,j,i} \Bigg) \end{aligned} \tag{13}$$

subject to (10)-(12). Rearranging (13), we have

$$\begin{aligned} Z_{\mathrm{LR}}(\boldsymbol{\lambda}) = \min & \sum_{i \in V} \sum_{\mathcal{R}_i \subseteq N_i} \mathrm{EMTX}_{i,\mathcal{R}_i} \cdot x_{i,\mathcal{R}_i} \\ & + \sum_{v \in D} \sum_{\langle i,j \rangle \in E} (\lambda_{v,i} - \lambda_{v,j}) e_{v,i,j} + \sum_{v \in D} (\lambda_{v,v} - \lambda_{v,s}). \end{aligned} \tag{14}$$

We observe that (10) and (14) imply that

$$e_{v,i,j} = \begin{cases} t_{i,j} & \text{if } \lambda_{v,i} - \lambda_{v,j} \leq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Thus, letting

$$u_{i,j} = \sum_{v \in D} \min(0, \lambda_{v,i} - \lambda_{v,j})$$

the optimal solution to the Lagrangian problem for a given $\boldsymbol{\lambda}$ reduces to finding

$$\begin{aligned} Z_{\mathrm{LR}}(\boldsymbol{\lambda}) = \min & \sum_{i \in V} \sum_{\mathcal{R}_i \subseteq N_i} \mathrm{EMTX}_{i,\mathcal{R}_i} \cdot x_{i,\mathcal{R}_i} \\ & + \sum_{\langle i,j \rangle \in E} u_{i,j} \cdot t_{i,j} + \sum_{v \in D} (\lambda_{v,v} - \lambda_{v,s}) \end{aligned} \tag{16}$$

subject to (11) and (12).

Now observe that (11), (12) and (16) imply that

$$t_{i,j} = \begin{cases} \sum_{\mathcal{R}_i:\mathcal{R}_i \subseteq N_i, j\in\mathcal{R}_i} x_{i,\mathcal{R}_i} & \text{if } u_{i,j} \leq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

Thus, letting

$$w_{i,j} = \min(0, u_{i,j})$$

the optimal solution to the Lagrangian problem for a given $\boldsymbol{\lambda}$ further reduces to finding

$$\begin{aligned} Z_{\mathrm{LR}}(\boldsymbol{\lambda}) = \min \sum_{i\in V}\sum_{\mathcal{R}_i\subseteq N_i}\Big(\mathrm{EMTX}_{i,\mathcal{R}_i} + \sum_{j\in\mathcal{R}_i} w_{i,j}\Big)x_{i,\mathcal{R}_i} \\ + \sum_{v\in D}\big(\lambda_{v,v} - \lambda_{v,s}\big) \end{aligned} \tag{18}$$

subject to (12).

This latter problem can be trivially solved by finding, for each $i \in V$, the subset $\mathcal{R}_i^* \subseteq N_i$ such that $\mathrm{EMTX}_{i,\mathcal{R}_i^*} + \sum_{j\in\mathcal{R}_i^*} w_{i,j}$ is the smallest among all $\mathcal{R}_i \subseteq N_i$. Then, we set

$$x_{i,\mathcal{R}_i} = \begin{cases} 1 & \text{if } \mathcal{R}_i = \mathcal{R}_i^* \text{ and } \mathrm{EMTX}_{i,\mathcal{R}_i^*} + \sum_{j\in\mathcal{R}_i^*} w_{i,j} < 0 \\ 0 & \text{otherwise.} \end{cases}$$

### 5.2 Lagrangian Heuristic

It is possible (though rare) that the solution to the Lagrangian problem for a given $\boldsymbol{\lambda}$ is also a feasible solution to the original problem. In such a case, since the dualized flow conservation constraints (7)-(9) are equalities, we know from the theory of Lagrangian relaxation that the solution is indeed an optimal solution to the original problem [35]. In general, solutions to the Lagrangian problem are infeasible solutions to the original problem. In such cases, it is useful to apply a judiciously designed Lagrangian heuristic for the infeasible solution to be made feasible [35].

Our design of the Lagrangian heuristic in this context readily utilizes the relaxation in the form of (18). For a given $\boldsymbol{\lambda}$, we define $\mathrm{EMTX}'$ as an amended EMTX metric for each $(i, \mathcal{R}_i)$ pair, $i \in V$, $\mathcal{R}_i \subseteq N_i$. From (18), we have

$$\mathrm{EMTX}'_{i,\mathcal{R}_i} = \mathrm{EMTX}_{i,\mathcal{R}_i} + \sum_{j\in\mathcal{R}_i} w_{i,j}. \tag{19}$$

A primal feasible solution is then obtained by applying the greedy algorithm presented in Section 6.1 using instead the $\mathrm{EMTX}'$ metric of (19). Effectively, the Lagrangian heuristic in this way aims to find a primal feasible solution that minimizes the objective of the Lagrangian problem.

### 5.3 Determining $\boldsymbol{\lambda}$

The best choice for $\boldsymbol{\lambda}$ is the optimal solution to the Lagrangian dual problem (denoted by LD), which is

$$Z_{\mathrm{LD}} = \max_{\boldsymbol{\lambda}} Z_{\mathrm{LR}}(\boldsymbol{\lambda}).$$

The subgradient method [36] is a popular technique for handling the Lagrangian dual problem. It starts by setting a lower bound $Z_{\mathrm{LB}} = -\infty$, an upper bound $Z_{\mathrm{UB}} = +\infty$, and choosing an initial value for $\boldsymbol{\lambda}$.

The general steps of the subgradient method in the $k$th iteration, $k = 0, 1, \ldots$, proceed as follows:

1) Given $\boldsymbol{\lambda}^{[k]}$, the set of Lagrange multipliers obtained in the $k$th iteration, we determine $Z_{\mathrm{LR}}(\boldsymbol{\lambda}^{[k]})$ by solving the Lagrangian problem for $\boldsymbol{\lambda}^{[k]}$. Then, we set $Z_{\mathrm{LB}} = Z_{\mathrm{LR}}(\boldsymbol{\lambda}^{[k]})$ if $Z_{\mathrm{LR}}(\boldsymbol{\lambda}^{[k]}) > Z_{\mathrm{LB}}$.
2) From $\{x_{i,\mathcal{R}_i} : i \in V, \mathcal{R}_i \subseteq N_i\}$, we obtain $\{t_{i,j} : \langle i,j\rangle \in E\}$ using (17), from which we obtain $\{e_{v,i,j} : v \in D, \langle i,j\rangle \in E\}$ using (15). If $\{e_{v,i,j} : v \in D, \langle i,j\rangle \in E\}$ satisfy (7), (8) and (9), we set $Z_{\mathrm{UB}} = Z_{\mathrm{LR}}(\boldsymbol{\lambda}^{[k]})$; otherwise, we apply the Lagrangian heuristic and obtain a primal feasible solution for $\boldsymbol{\lambda}^{[k]}$. In the latter case, we compute the sum of EMTX over all internal nodes of the multicast tree. If the result is smaller than $Z_{\mathrm{UB}}$, we update $Z_{\mathrm{UB}}$ accordingly.
3) Calculate $\delta^{[k]}$, the step size in the $k$th iteration, by

$$\delta^{[k]} = \frac{\pi\left[Z_{\mathrm{UB}} - Z_{\mathrm{LR}}(\boldsymbol{\lambda}^{[k]})\right]}{\|\boldsymbol{\phi}^{[k]}\|^2}, \tag{20}$$

where $\pi$ is a scalar satisfying $0 < \pi \leq 2$, $\|\cdot\|$ denotes the euclidean norm, and $\boldsymbol{\phi}^{[k]} = \{\phi_{v,i}^{[k]} : v \in D, i \in V\}$ is the subgradient vector in the $k$th iteration given by

$$\phi_{v,s}^{[k]} = \sum_{j:\langle s,j\rangle\in E} e_{v,s,j}^{[k]} - \sum_{j:\langle j,s\rangle\in E} e_{v,j,s}^{[k]} - 1$$

$$\phi_{v,v}^{[k]} = \sum_{j:\langle v,j\rangle\in E} e_{v,v,j}^{[k]} - \sum_{j:\langle j,v\rangle\in E} e_{v,j,v}^{[k]} + 1$$

$$\phi_{v,i}^{[k]} = \sum_{j:\langle i,j\rangle\in E} e_{v,i,j}^{[k]} - \sum_{j:\langle j,i\rangle\in E} e_{v,j,i}^{[k]}, \quad i \in V - \{s, v\}.$$

4) Obtain the set of Lagrange multipliers $\boldsymbol{\lambda}^{[k+1]}$ in the $(k+1)$th iteration as

$$\lambda_{v,i}^{[k+1]} = \lambda_{v,i}^{[k]} + \delta^{[k]}\phi_{v,i}^{[k]}, \quad v \in D, \quad i \in V.$$

Steps 1) to 4) are repeated until the method reaches a specified termination criterion.

In all experiments where we solve the EMTX-based multicast problem using Lagrangian relaxation, we set $\lambda_{v,i}^{[0]} = 0$, $v \in D$, $i \in V$. In choosing a value for $\pi$ in (20), we follow the approach of [36]. We let $\pi = 2$ for $2|V|$ iterations and then successively halve both the value of $\pi$ and the number of iterations until the number of iterations reaches a threshold value of five. Then, we halve the value of $\pi$ every five iterations. The subgradient method is terminated either when $Z_{\mathrm{LB}} = Z_{\mathrm{UB}}$ or when $(Z_{\mathrm{UB}} - Z_{\mathrm{LB}})/Z_{\mathrm{UB}} < 10\%$. In the case where $Z_{\mathrm{LB}} = Z_{\mathrm{UB}}$, we have an optimal solution to the original problem.

## 6 Centralized Algorithm

It is known that the set cover problem cannot be approximated to within less than a logarithmic factor [37]. The fact that the set cover problem is polynomial-time reducible to the EMTX-based multicast problem implies that we cannot expect to solve our problem in polynomial time with an approximation ratio better than $\mathcal{O}(\ln|\mathcal{D}|)$. We have shown in [38] that the EMTX-based multicast problem can be transformed into a node-weighted directed Steiner tree problem. That approach yields a polynomial-time solution with an approximation ratio of $\mathcal{O}(4\ln|\mathcal{D}|)$. However, it requires transformation of the network graph $G$ into an auxiliary graph, and therefore makes it impossible to be implemented in a distributed fashion. In this section, we propose a greedy algorithm for tackling the EMTX-based multicast problem. Later, in Section 7, we show how this centralized algorithm can be extended to a distributed algorithm.

### 6.1 Greedy Algorithm

The algorithm starts with an initial tree $T$ including only the source node $s$. At every step of the tree-building process, for each destination node $v$ in the set $\mathcal{D}$ that is not yet included in $T$, we find the directed path requiring minimum cost among all shortest paths from nodes in $T$ to the destination node $v$. We identify among all $v$ the destination node $v^*$ whose corresponding path has the smallest cost, where ties can be broken arbitrarily. Then, we add node $v^*$ and its associated path to $T$, and, for each directed link $\langle i,j\rangle$ in the path, we add node $j$ as a next-hop receiver of node $i$ in $T$. When these are done, we update the set $\mathcal{D}$ by removing $v^*$ from $\mathcal{D}$. The process continues until $\mathcal{D}=\emptyset$ (empty set), meaning all destination nodes have been included in $T$ and we have obtained a complete $T$ based on the greedy algorithm.

For the purpose of EMTX-based multicast routing, we define the cost of a path in this context as the sum of *additional* EMTX required by the forwarding nodes in the sequence of directed links along the path. The concept of additional EMTX can be conveniently explained by using the example provided in Fig. 1.

Consider node $s$ as the source node, both node $u$ and node $v$ as members of the multicast group. Since the initial $T$ includes node $s$ only, adding node $u$ to $T$ would incur an additional EMTX at the sending node $s$ given by $\mathrm{EMTX}_{s,\{u\}}=1.25$, while for node $v$ the additional EMTX required at node $s$ would be $\mathrm{EMTX}_{s,\{v\}}=1.43$. The greedy algorithm thus chooses node $u$ as the first destination node to be included in $T$. Now, for node $v$, it has two choices:

(1) Using the directed path formed by the directed link $\langle u,v\rangle$ would incur an additional EMTX at the sending node $u$ given by $\mathrm{EMTX}_{u,\{v\}}=1.67$.

(2) Using the directed path formed by the directed link $\langle s,v\rangle$ would incur an additional EMTX at the sending node $s$ given by $\mathrm{EMTX}_{s,\{u,v\}}-\mathrm{EMTX}_{s,\{u\}}=1.61-1.25=0.36$. The calculation of the additional EMTX in this form for this choice is simply because node $u$ has already been included in $T$ as a next-hop receiver of node $s$. Thus, by exploiting the wireless broadcast advantage, a multicast transmission from node $s$ to both node $u$ and node $v$ requires no more than an EMTX of 1.61.

The greedy algorithm thus chooses the directed path $\{\langle s,v\rangle\}$ for node $v$, and it turns out to be the optimal solution to this particular problem instance.

For ease of calculating the additional EMTX for path selection, at the beginning of the algorithm, we initialize the weight of each directed link $\langle i,j\rangle$ to $\mathrm{EMTX}_{i,\{j\}}$. Then, at every step after the selected destination node and its associated path are included in $T$, we dynamically adjust the weight of each relevant directed link. Specifically, for each directed link $\langle i,j\rangle$ in the path, node $i$ is included in $T$ as a forwarding node, and node $j$ is included in $T$ as a next-hop receiver of node $i$. Thus, for each node $n$ in the single-hop neighborhood $N_i$ of node $i$ but not in $T$, we adjust the weight of the directed link $\langle i,n\rangle$ to $\mathrm{EMTX}_{i,\mathcal{R}_i+\{n\}}-\mathrm{EMTX}_{i,\mathcal{R}_i}$, where $\mathcal{R}_i$ is the set of next-hop receivers of node $i$ currently in $T$. This is due to the fact that for any sending node $i$ in $T$, at every step of the greedy algorithm, at most one additional node in its single-hop neighborhood can be added as its next-hop receiver.

Let $\mathrm{WEIGHT}_{i,j}$ denote the weight of the directed link $\langle i,j\rangle$. Let $\mathrm{MACP}(T,v)$ denote the directed path requiring minimum cost among all shortest paths from nodes in $T$ to the destination node $v$ not in $T$. Let $\mathrm{COST}(T,v)$ denote the cost of $\mathrm{MACP}(T,v)$. By definition, we have

$$\mathrm{COST}(T,v)=\sum_{\langle i,j\rangle\in\mathrm{MACP}(T,v)}\mathrm{WEIGHT}_{i,j}.$$

A pseudo-code of the greedy algorithm is described below in Algorithm 1.

**Algorithm 1.** Greedy Algorithm

**Input:** $G=(V,E)$, $s$, $\mathcal{D}$, $\{p_{i,j}\}$
$T\leftarrow\{s\}$
  **for all** $i\in V$ **do**
  $\mathcal{R}_i\leftarrow\emptyset$
**end for**
**for all** $\langle i,j\rangle\in E$ **do**
  $\mathrm{WEIGHT}_{i,j}\leftarrow\mathrm{EMTX}_{i,\{j\}}$
**end for**
**while** $\mathcal{D}\neq\emptyset$ **do**
  **for all** $v\in\mathcal{D}$ **do**
    Find $\mathrm{MACP}(T,v)$ and $\mathrm{COST}(T,v)$
  **end for**
  Find $v^*=\arg\min_v\mathrm{COST}(T,v)$
  $T\leftarrow T+\mathrm{MACP}(T,v^*)$
  **for all** $\langle i,j\rangle\in\mathrm{MACP}(T,v^*)$ **do**
    $\mathcal{R}_i\leftarrow\mathcal{R}_i+\{j\}$
    **for all** $n\in N_i-\mathcal{R}_i$ **do**
      $\mathrm{WEIGHT}_{i,n}\leftarrow\mathrm{EMTX}_{i,\mathcal{R}_i+\{n\}}-\mathrm{EMTX}_{i,\mathcal{R}_i}$
    **end for**
  **end for**
  $\mathcal{D}\leftarrow\mathcal{D}-\{v^*\}$
**end while**

We note that this algorithm requires at most $\mathcal{O}(|\mathcal{D}||V|^3)$ time, since finding $\mathrm{MACP}(T,v)$ for all $v$ can be completed in

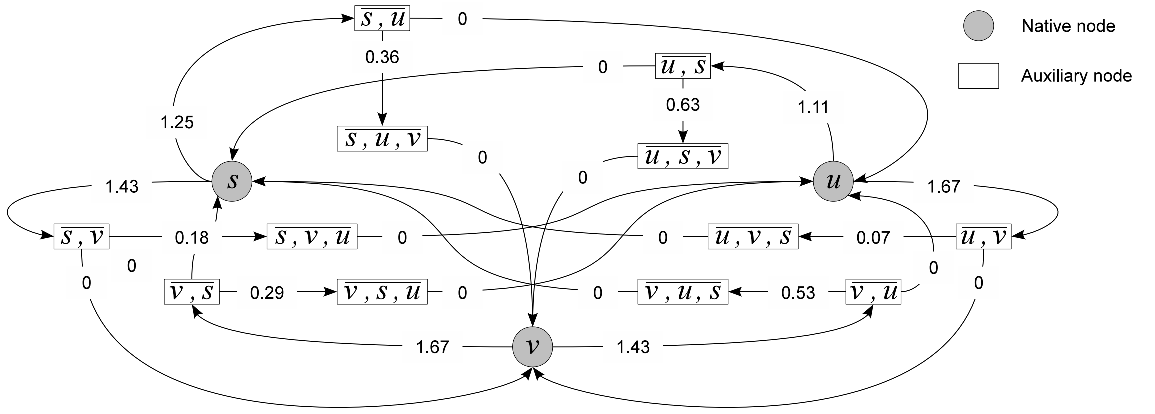



Fig. 3. Auxiliary graph of the three-node wireless mesh network depicted in Fig. 1.

at most $\mathcal{O}(|V|^3)$ time by applying Dijkstra's shortest path algorithm [39] at each origin in $T$ and hence for up to $|V|$ times.

## 6.2 Approximation Ratio

The proposed greedy algorithm for EMTX-based multicast routing requires updating the link weights at each step to reflect the incremental cost in EMTX. This makes it difficult to analyze the performance of the algorithm with respect to the global optimum. We describe below a way to transform the network graph $G$ into an auxiliary graph $G'$ that incorporates all information on the incremental cost in EMTX. Such a self-contained auxiliary graph allows us to investigate the approximation ratio of the algorithm using known results in the literature.

We begin by including in $G'$ all nodes in $V$ of the original graph $G$. For convenience, we call such nodes as *native nodes* in the auxiliary graph $G'$. Then, for each $(i, N_i)$ pair in the original graph $G$, we enumerate the set of ordered choices of $|N_i|$ elements from $N_i$. For each such ordered choice of $|N_i|$ elements $\{j_1, j_2, \ldots, j_{|N_i|}\}$, where $j_1, j_2, \ldots, j_{|N_i|} \in N_i$, we add $|N_i|$ new nodes, called *auxiliary nodes*, into the auxiliary graph $G'$ one by one in the following sequence:

- At step 1, we use the element $j_1$ and accordingly add a Level-1 auxiliary node labelled as $\overline{i, j_1}$. We connect the native node $i$ and the auxiliary node $\overline{i, j_1}$ in $G'$ with a directed link from node $i$ to node $\overline{i, j_1}$; the weight of the link is set to $\mathrm{EMTX}_{i,\{j_1\}}$. Then, we connect the auxiliary node $\overline{i, j_1}$ and the native node $j_1$ in $G'$ with a directed link from node $\overline{i, j_1}$ to node $j_1$; the weight of the link is set to 0.
- At each subsequent step $k$, $k = 2, 3, \ldots, |N_i|$, we use the element $j_k$ and accordingly add a Level-$k$ auxiliary node labelled as $\overline{i, j_1, \ldots, j_{k-1}, j_k}$. We connect the auxiliary node $\overline{i, j_1, \ldots, j_{k-2}, j_{k-1}}$ (added in step $k-1$) and the auxiliary node $\overline{i, j_1, \ldots, j_{k-1}, j_k}$ in $G'$ with a directed link from node $\overline{i, j_1, \ldots, j_{k-2}, j_{k-1}}$ to node $\overline{i, j_1, \ldots, j_{k-1}, j_k}$; the weight of the link is set to

$$\mathrm{EMTX}_{i,\{j_1,\ldots,j_{k-1},j_k\}} - \mathrm{EMTX}_{i,\{j_1,\ldots,j_{k-2},j_{k-1}\}}.$$

  Then, we connect the node $\overline{i, j_1, \ldots, j_{k-1}, j_k}$ and the native node $j_k$ in $G'$ with a directed link from node $\overline{i, j_1, \ldots, j_{k-1}, j_k}$ to node $j_k$; the weight of the link is set to 0.

On the auxiliary graph $G'$ constructed in this way, the EMTX-based multicast problem for a multicast session $(s, \mathcal{D})$ on the original graph $G$ reduces to a directed Steiner tree problem [40]. The directed Steiner tree problem on $G'$ is to find the minimum cost directed tree rooted at the native node $s$ and spanning all the native nodes in $\mathcal{D}$.

As an example, Fig. 3 depicts the auxiliary graph constructed for the three-node wireless mesh network shown in Fig. 1. The mesh network in Fig. 1 consists of three nodes $s$, $u$ and $v$. These nodes become the native nodes in the auxiliary graph in Fig. 3. Note that the native nodes are shown as shaded circles in the figure. All the other nodes in the auxiliary graph, which take the shape of a rectangle in Fig. 3, are the auxiliary nodes. Consider the link connecting the native node $s$ and the Level-1 auxiliary node $\overline{s, u}$, the cost of this link is 1.25 which is the EMTX for $s$ to reach the node set $\{u\}$ or $\mathrm{EMTX}_{s,\{u\}}$. Now, consider the link connecting $\overline{s, u}$ and the Level-2 auxiliary node $\overline{s, u, v}$. The cost of this link is the incremental cost for the node $s$ to multicast to both nodes $u$ and $v$ if node $s$ is already multicasting to $u$. Hence the cost is

$$\mathrm{EMTX}_{s,\{u,v\}} - \mathrm{EMTX}_{s,\{u\}} = 1.61 - 1.25 = 0.36.$$

Note that the cost of the edge from an auxiliary node preceding a native node to the corresponding native node is always zero.

We now consider again the multicast session $(s, \mathcal{D} = \{u, v\})$. We recall in Fig. 1 that the optimal solution to the EMTX-based multicast problem is the tree formed by links $\langle s, u\rangle$ and $\langle s, v\rangle$, which requires an EMTX of 1.61. In Fig. 3, one can observe that the optimal solution to the directed Steiner tree problem is the tree formed by links $\langle s, \overline{s, u}\rangle$, $\langle \overline{s, u}, u\rangle$, $\langle \overline{s, u}, \overline{s, u, v}\rangle$ and $\langle \overline{s, u, v}, v\rangle$, which yields a cost of 1.61. One may also observe that the tree formed by links $\langle s, \overline{s, v}\rangle$, $\langle \overline{s, v}, v\rangle$, $\langle \overline{s, v}, \overline{s, v, u}\rangle$ and $\langle \overline{s, v, u}, u\rangle$ yields the same minimum cost, which represents essentially the same optimal solution on the original graph $G$.

We see that, for a multicast session $(s, \mathcal{D})$ on such an auxiliary graph, the greedy algorithm we proposed for EMTX-based multicast routing has the same effect as the minimum-cost path heuristic described by Ramanathan [41] for the directed Steiner tree problem. Starting with the tree $T$ consisting the source node $s$ only, at each step in this algorithm, a new node in $\mathcal{D}$ is included together with a minimum-cost path extended from the tree to the node. Ramanathan showed that, for networks with asymmetric links, the approximation ratio of the minimum-cost path heuristic is guaranteed by a ratio proportional to how asymmetric the graph can be. Here, we contribute a theorem showing that the worst-case approximation ratio of our proposed greedy algorithm depends on the maximum out-degree of the optimal EMTX-based multicast tree and the asymmetry of links in the graph with respect to EMTX.

Let $T_{\text{opt}}$ denote the optimal EMTX-based multicast tree on the original graph $G$. By definition, $\mathcal{I}(T_{\text{opt}})$ is the set of internal nodes of $T_{\text{opt}}$, and, for each forwarding node $i$ in the set $\mathcal{I}(T_{\text{opt}})$, $\mathcal{R}_i$ is the set of its next-hop receivers in $T_{\text{opt}}$. Let $d_{\max,\text{out}}$ denote the maximum out-degree of $T_{\text{opt}}$, i.e.,

$$d_{\max,\text{out}} = \max_{i \in \mathcal{I}(T_{\text{opt}})} |\mathcal{R}_i|.$$

Define

$$\Psi(T_{\text{opt}}) = \max_{(i,j): i \in \mathcal{I}(T_{\text{opt}}), j \in \mathcal{R}_i} \frac{\text{EMTX}_{j,\{i\}}}{\text{EMTX}_{i,\{j\}}}.$$

**Theorem 8.** *The greedy algorithm for the EMTX-based multicast problem has a worst-case approximation ratio of* $[1 + \Psi(T_{\text{opt}})] d_{\max,\text{out}}$.

**Proof.** See Appendix H available in the online supplemental material. □

## 7 Distributed Algorithm

This section describes the distributed implementation of our centralized algorithm for EMTX-based multicast routing. In practice, members of the multicast group are likely to join the multicast session at different time. The principle of our design of the distributed algorithm is thus for each new member of the multicast group to initiate the procedure for finding a directed path from the existing tree. In particular, the new destination node chooses the path with the minimum sum of additional EMTX required. Again, the algorithm exploits the wireless broadcast advantage at the point where the branch to the node is extended from the existing tree.

For convenience, we call any node in the existing tree as a *session member*. The algorithm requires each session member to maintain the set of its upstream nodes towards the source node and the set of its next-hop receivers in the current tree. For node $i$ in the current tree, let $\mathcal{P}_i$ be the set of its upstream nodes and $\mathcal{R}_i$ be the set of its next-hop receivers. By definition, $\mathcal{P}_i = \emptyset$ if node $i$ is the source node; $\mathcal{R}_i = \emptyset$ if node $i$ is a leaf node of the tree.

### 7.1 Node Join

When node $v$ wishes to join the multicast session as a destination, it broadcasts a `Join_Req` message. The `Join_Req` message contains the information about the multicast group address, the IP address of node $v$, the sequence number, and the path cost (initially set to zero).

If a node that is not a session member receives a `Join_Req` message for node $v$, it broadcasts the `Join_Req` message to its single-hop neighbors. Before broadcasting the message, the node updates the path cost by adding the additional EMTX of its link to the incoming node. The node then marks the incoming node as its reverse entry to node $v$. In cases where the node receives multiple `Join_Req` messages for node $v$ from its single-hop neighbors, it broadcasts each such message and updates its reverse entry accordingly so long as the updated cost indicates a shorter path to node $v$.

If a session member receives a `Join_Req` message for node $v$, it instead replies with a `Join_Reply` message. The `Join_Reply` message contains the path cost from the session member to node $v$, obtained by updating the path cost retrieved from the `Join_Req` message. Since the session member may receive multiple `Join_Req` messages for node $v$ from its single-hop neighbors, it replies only after a timeout period (500 msec in our implementation) and chooses the incoming node with the smallest updated path cost as its reverse entry to node $v$. The `Join_Reply` message is unicast all the way back towards node $v$, using the reverse entry kept at each intermediate node along the path.

Each node that is not a session member may receive multiple `Join_Reply` messages for node $v$. In such cases, it forwards each such message so long as the message indicates a shorter path to node $v$. It also updates the incoming node as its forward entry to the corresponding session member that initiates the `Join_Reply` message. When node $v$ receives multiple `Join_Reply` messages, it chooses the one that indicates the shortest path. Then, it unicasts a `Route_Activate` message all the way towards the nominated session member, using the forward entry kept at each intermediate node along the path. The route is activated by setting the intermediate nodes as forwarding nodes in the updated multicast tree. Each node $i$ along the path updates the set $\mathcal{P}_i$ of its upstream nodes and the set $\mathcal{R}_i$ of its next-hop receivers accordingly.

### 7.2 Node Departure

When a destination node $v$ wishes to leave the multicast session, it is required to check whether it is currently a forwarding node in the multicast tree. If so, node $v$ will stay in the multicast session; otherwise, it sends a `Prune` message to its parent node and removes itself from the multicast tree. When a forwarding node $i$ receives a `Prune` message from a next-hop receiver, if node $i$ is a destination node or has multiple next-hop receivers in the multicast tree, it simply deletes the node from the set $\mathcal{R}_i$ of its next-hop receivers and remains in the multicast tree; otherwise, it forwards the `Prune` message to its parent node (if there is) and removes itself from the multicast tree.

### 7.3 Tree Repair

When a forwarding node $i$ fails in the network, its next-hop receivers in the set $\mathcal{R}_i$ are responsible for repairing the multicast tree. Each node in the set $\mathcal{R}_i$ initiates the repair process by broadcasting a `Repair_Req` message. If a non-session member receives a `Repair_Req` message, it broadcasts the `Repair_Req` message to its neighbors using the same treatment for a `Join_Req` message as described in Section 7.1. If a session member who is an upstream node of node $i$ receives a `Repair_Req` message, it replies with a `Repair_Reply` message using the same treatment for a `Join_Reply` message as described in Section 7.1. However, in the case where a session member whose upstream nodes include node $i$ receives a `Repair_Req` message, since the session member itself is disconnected in the disrupted tree, it is not allowed to reply but broadcasts the `Repair_Req` message to its neighbors using the same treatment for a `Join_Req` message. Similar to the joining process described in Section 7.1, each node in the set $\mathcal{R}_i$ may receive multiple `Repair_Reply` messages in the repair process. In all cases, it chooses the repair path with the minimum sum of additional EMTX required.

## 8 Performance Evaluation

This section provides detailed numerical results that we have obtained for evaluating the performance of EMTX as a routing metric for reliable multicast in multi-hop wireless mesh networks.

### 8.1 Protocol Implementation

We modify the MAC layer of IEEE 802.11 by using RMAC [10] rather than the default CSMA/CA for multicast. The ARQ mechanism of RMAC uses *busy tone* to realize MAC-layer multicast reliability. Using a variable-length control frame, RMAC stipulates the response order of receivers to resolve feedback collision.

We apply the probing technique of [18] to measure the link quality required for EMTX calculation. Node $i$ broadcasts a probe that contains 134 bytes of payload at every second. Each probe sent by node $i$ also contains the number of probes received by node $i$ from each of its single-hop neighbors during the last 10 seconds. For every $(i, j)$ pair, this technique allows node $i$ to estimate the forward delivery probability $\overrightarrow{d_{i,j}}$ for data frames sent to node $j$ and the reverse delivery probability $\overleftarrow{d_{i,j}}$ for ACK frames received from node $j$.

### 8.2 Simulation Setup

We use QualNet [42] to simulate a network with 50 mesh routers. The nodes are uniformly distributed in an area of size $1,500 \times 1,500$ m. Each node has one interface, working in IEEE 802.11b. All experiments use the two-ray propagation path loss model, with free space path loss exponent of 2 for near sight and plane earth path loss exponent of 4 for far sight. The maximum number of MAC-layer retransmissions is set to seven for each packet at each node.

In each experiment, we set up one multicast constant bit rate (MCBR) session from the source node $s$ to the set of destination nodes $\mathcal{D}$. The size of each multicast packet is 512 bytes. Two bit rates are considered for the MCBR traffic: 100 Kbps as the low traffic load and 400 Kbps as the high traffic load. Each forwarding node in the MCBR session buffers the incoming packets and schedules them for MAC-layer transmissions in a first-in-first-out fashion. Background traffic in the form of unicast flows is randomly generated to increase the chance of packet collision. Ten different topologies are randomly generated from QualNet. The size of the multicast group, which is $|\mathcal{D}| + 1$ in this context, is varied from 5 to 45 at an interval of 5. For a given group size and topology, 10 different $(s, \mathcal{D})$ pairs are selected. The total simulation time in each experiment is 40 minutes.

### 8.3 Approximation Ratio

The analysis in Section 2 shows that, for a multicast session $(s, \mathcal{D})$ on the network graph $G$, the worst-case approximation ratio of the proposed greedy algorithm depends on the maximum out-degree of the optimal EMTX-based multicast tree and the asymmetry of links in the graph with respect to EMTX. On the other hand, we know that no polynomial-time algorithm can solve the EMTX-based multicast problem with an approximation ratio better than $\mathcal{O}(\ln|\mathcal{D}|)$. Here, we are interested in investigating the empirical performance of the centralized algorithm.

For this purpose, in each experiment described in this paper, we collect the link quality after QualNet has been run for 10 minutes with background traffic only. For each graph thus obtained and for the specific $(s, \mathcal{D})$ pair, we solve the EMTX-based multicast problem using Lagrangian relaxation presented in Section 5. The approximation ratio of the centralized algorithm is obtained as the ratio of the result of the centralized algorithm to $Z_{\text{LB}}$. Note that $Z_{\text{LB}}$ is a lower bound on the optimal value of the original problem. Therefore, the actual approximation ratio of the centralized algorithm is likely to be smaller in those cases where $Z_{\text{LB}} < Z_{\text{UB}}$.

Fig. 4a demonstrates the worst-case performance of the centralized algorithm (CA for short in the legend) in each particular choice of the multicast group size. The average results over all corresponding experiments are also shown in the figure. Note that the dashed line with the legend "$\ln|\mathcal{D}|$" indicates where the theoretical limit stands in the figure. We observe that most of the results are indeed well below $\ln|\mathcal{D}|$. This is especially true when $|\mathcal{D}|$ is large, representing a large multicast group, which is the case where reducing the number of multicast transmissions becomes important. For example, when the group size is 45 and hence $|\mathcal{D}| = 44$, the worst-case approximation ratio of any polynomial-time algorithm for this problem can be as bad as $\ln 44 = 3.8$ in theory. The performance of the proposed greedy algorithm is about a factor of 2.3 in the worst case and 1.5 on average in our experiments.

We also include in Fig. 4a the results of the distributed algorithm (DA for short in the legend), considering node join only, for each particular choice of the multicast group size. The performance of the distributed algorithm in this case depends on the joining sequence of the group members in forming the multicast tree. We randomly generate 10 different joining sequences for each $(s, \mathcal{D})$ pair in our experiments. The results suggest that the distributed algorithm is comparable with the centralized algorithm in the average performance. Although the worst-case performance of the distributed algorithm becomes poor when the group size is large, it is still below $\ln|\mathcal{D}|$.

### 8.4 Effectiveness of EMTX-Based Multicast Routing

We further demonstrate the effectiveness of EMTX-based multicast routing by comparing it with the two baseline

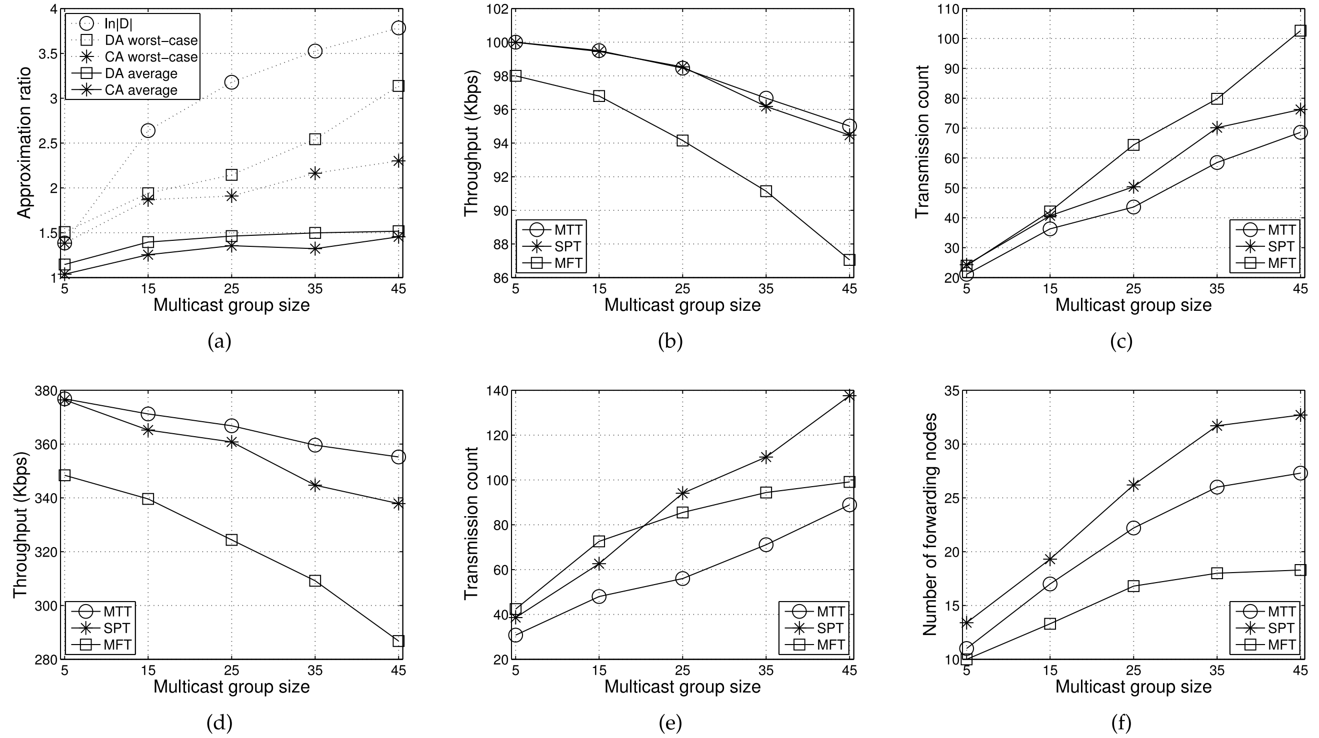



Fig. 4. Numerical results. (a) Approximation ratio of centralized and distributed algorithms for the EMTX-based multicast problem. (b) Throughput of the 100 Kbps MCBR traffic. (c) Transmission count of the 100 Kbps MCBR traffic. (d) Throughput of the 400 Kbps MCBR traffic. (e) Transmission count of the 400 Kbps MCBR traffic. (f) Comparison of the number of forwarding nodes.

approaches, SPT and MFT, as discussed in Section 2. For ease of description, we shall hence call our EMTX-based multicast approach as the minimum transmission tree (MTT).

Recall that SPT forms the multicast tree by finding a shortest path from the source node to each destination node using the high-throughput ETX metric [18]. As a result, SPT in this context takes the link quality into account but not the wireless broadcast advantage.

MFT aims instead to find the multicast tree that minimizes the number of forwarding nodes [19]. This approach implicitly assumes that all links in the wireless network are 100 percent reliable. In this way, MFT makes use of the wireless broadcast advantage but is ignorant of the link quality. MFT is a special case of MTT. In the case where all $p_{i,j}$ values in the network are one, each node requires no more than one multicast transmission for a packet to be successfully received and acknowledged by its next-hop receivers, if the node is included in the multicast tree as a forwarding node. MFT was shown in [19] to be an NP-hard problem. When all $p_{i,j}$ values in the network are set to one, our proposed distributed algorithm for EMTX-based multicast routing reduces to a distributed implementation of MFT.

In all experiments where we use the two baseline approaches in forming the multicast tree, we obtain the actual multicast routing performance with the probabilistic packet reception model in the case of MFT and that with the wireless broadcast advantage in the case of SPT. The simulation-based study of the multicast routing performance is focused on two important performance measures:

- *Throughput*, defined as the average rate of successful end-to-end packet delivery for the multicast session, measured in bits per second. A higher throughput in this context is equivalent to a higher end-to-end packet delivery ratio of the multicast session.
- *Transmission count*, defined as the average number of multicast transmissions (including retransmissions) required for end-to-end delivery of each packet in the multicast session. A smaller transmission count indicates less consumption of the network bandwidth and less interference to other users of the same spectrum, which is desirable in multi-hop wireless networking.

Figs. 4b and 4c compare the multicast routing performance for the low traffic load. Figs. 4d and 4e provide the results for the high traffic load. We observe in both cases that, when the multicast group size increases, the throughput decreases yet at the expense of increasing transmission count. This is because a larger size of the multicast session implies that a larger number of forwarding nodes is required. This effect is demonstrated in Fig. 4f where we show the average number of forwarding nodes required for each multicast algorithm in comparison as the group size increases. This in general results in an increased number of multicast transmissions, essentially increasing the likelihood of packet collisions. Since the maximum number of MAC-layer retransmissions is set to seven, this effect increases the likelihood of packet loss when the retransmission limit is reached. This is why the throughput drops as the multicast group size increases.

The results demonstrate that MTT outperforms SPT. MTT uses EMTX as the routing metric and hence utilizes the wireless broadcast advantage. However, SPT is unable to use this feature since ETX is a unicast metric. Therefore, SPT results in more forwarding nodes in the multicast tree and hence more transmissions to deliver packets, which increases the possibility of collision-induced packet loss. This happens for both low and high traffic loads. Since both MTT and SPT use good-quality links, both can achieve high end-to-end throughput. However, because SPT cannot exploit the benefit of broadcast advantage, it requires many more transmissions to achieve the comparable throughput with MTT. As a result, MTT can save more available network resource for other traffic in the network.

On the other hand, although MFT can significantly reduce the number of forwarding nodes due to the nature of its design, ignoring link qualities makes it likely to use bad-quality links which leads to significant number of retransmissions without making successful end-to-end progress. We observe in Fig. 4e that, when the traffic load is high and the group size is large, MFT requires significantly smaller transmission count than SPT. This is because in such scenarios the chance of collision-induced packet loss is high. In the case of MFT where bad-quality links are likely to be used, the chance of a packet drop in the early stage of the multi-hop transmission is also high. This effect results in a significantly reduced transmission count, along with a significantly reduced throughput as demonstrated in Fig. 4d.

In both cases, we see that EMTX-based multicast routing requires on average much smaller transmission count and yet achieves higher throughput. In particular, the transmission count of MTT is up to 35 percent smaller than that of MFT and 40 percent smaller than that of SPT. The throughput of MTT is up to 24 percent higher than that of MFT and 5 percent higher than that of SPT. This is achieved by MTT striking a balance between the two extremes, and thus effectively captures the combined effects of wireless broadcast advantage and link quality awareness.

## 9 CONCLUSION

Our focus in this paper is on developing high-throughput algorithms for reliable multicast routing in multi-hop wireless mesh networks. To address this challenge, we have proposed EMTX as a robust metric that captures the combined effects of MAC-layer retransmission-based reliability, wireless broadcast advantage, and link quality awareness. We have formulated the EMTX-based multicast problem with the objective of minimizing the sum of EMTX over all forwarding nodes in the multicast tree. Both centralized and distributed algorithms have been designed for the multicast problem. We have also implemented the distributed algorithm as a multicast routing protocol. Extensive simulation experiments have confirmed that, compared to two baseline approaches, EMTX-based multicast routing can effectively reduce transmission overhead and yet enhance multicast throughput. Open research problems include studying the performance of the proposed protocol in more realistic simulation environments as well as real-life wireless networks.

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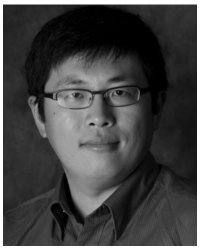
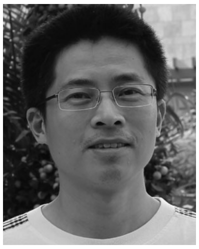


**Xin Zhao** received the BS degree in electronics and the MEng degree in telecommunication from Wuhan University, China, in 2000 and 2003, respectively, and the PhD degree in computer science from the University of New South Wales, Australia, in 2010. From 2010 to 2012, he was a research associate in the School of Computer Science and Engineering, The University of New South Wales. He is currently a software engineer in the Australian Centre for Field Robotics, The University of Sydney, Australia. He is a member of the IEEE.



**Jun Guo** received the BE degree in automatic control engineering from the Shanghai University of Science and Technology, China, in 1992, the ME degree in telecommunications engineering, and the PhD degree in electrical and electronic engineering from the University of Melbourne, Australia, in 2001 and 2006, respectively. He was with the School of Computer Science and Engineering, University of New South Wales, Australia, as a senior research associate from 2006 to 2008 and on an Australian Postdoctoral Fellowship supported by the Australian Research Council from 2009 to 2011. Since March 2012, he has been with the Department of Electronic Engineering, City University of Hong Kong, where he is currently a visiting assistant professor. His current research interests are survivable network topology design and teletraffic theory and its applications in service sectors. He is a member of the IEEE.

**Chun Tung Chou** received the BA degree in engineering science from the University of Oxford, United Kingdom, and the PhD degree in control engineering from the University of Cambridge, United Kingdom. He is currently an associate professor at the School of Computer Science and Engineering, University of New South Wales, Australia. His current research interests are wireless sensor networks, compressive sensing and nano-communication. He is a member of the IEEE.

**Archan Misra** received the BTech degree in electronics and communication engineering from IIT Kharagpur, India, and the PhD degree in electrical and computer engineering from the University of Maryland at College Park. He is currently an associate professor of information systems at Singapore Management University, and a director of the LiveLabs research center. His broad research interests are in the areas of pervasive computing and mobile systems, including energy-efficient mobile analytics and activity recognition. Over the past 12 years, as part of his previous employment with IBM Research and Telcordia Technologies, he has published extensively in the areas of wireless networking and pervasive computing, and is a co-author on papers that received the Best Paper awards from EUC 2008, ACM WOWMOM 2002, and IEEE MILCOM 2001. He is presently an editor of the *IEEE Transactions on Mobile Computing* and the *Journal of Pervasive and Mobile Computing* and chaired the IEEE Computer Society's Technical Committee on Computer Communications from 2005–2007. He is a member of the IEEE.

**Sanjay K. Jha** is a professor and the head of the Network Group at the School of Computer Science and Engineering, University of New South Wales. His research activities cover a wide range of topics in networking including wireless sensor networks, adhoc/community wireless networks, resilience and multicasting in IP networks, and security protocols for wired/wireless networks. He has published more than 160 articles in high quality journals and conferences. He is the principal author of the book *Engineering Internet QoS* and a co-editor of the book *Wireless Sensor Networks: A Systems Perspective*. He was an associate editor of the *IEEE Transactions on Mobile Computing*. He is currently on the editorial board of the *ACM Computer Communication Review*. He is a senior member of the IEEE.

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